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MATHEMATICS, GAMES AND ECONOMIC BEHAVIOR

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FOREWORD

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MATHEMATICS, GAMES AND ECONOMIC BEHAVIOR

USSR

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Every practical application of mathematics to the analysis of this or that sector of factual events no longer appertains to mathematics itself, but to that science which investigates the existing phenomena. Therefore the use of mathematics in the social sciences — in particular in economics which is an organic part of the social sciences — acquires traits which are peculiar to the latter. To believe that as a result of the attraction of mathematics the formulation and development of argumentation of this or that thesis in the sphere of social sciences acquires the characteristics of a higher class truth, means to uproot the principle of party-mindedness which is compulsory in social sciences for any form of argumentation, for the analysis of the quality of social phenomena, as well as for the analysis of their quantitative correlations which are indissolubly linked with them.

What has been mentioned here also fully applies to the application of one of the recently worked out new sectors in mathematics — the theory of games, more specifically of strategic games — in economics. This question deserves extensive attention, since voices can be heard among bourgeois economists about the specifically important and central meaning of the theory of game for the methodology of political economics. They compare its role in the social sciences with the role of geometry in physics. There is talk about a new American school which bases its research on the theory of games as an important methodological principle, etc.

It is well known that if some mathematical methods are used in false, apologetic formulations, it does not follow from this that the corresponding theorems of mathematics in themselves are incorrect; either the initial analysis is erroneous, or the premise of the results arrived at, the interpretation of the meaning of the theorems used, and not infrequently the application of the theorems are incorrect.

No one needs to have doubt about the well-known rules of the findings of maximum-minimum functions of many variables simply because these rules are called upon in the capacity of the mathematical apparatus for the formulation of the basic theses of the theory of limited utility; and no one needs to have doubt about the correctness of the effective application in radio and other spheres of technology of the theory of quasi-periodical variation because it is being utilized by apologists in attempts to give a false explanation of the boom-and-bust cycle in the capitalist economy. And if it happened that the contemporary theory of games, which in 1928 grew out of one of the works of the outstanding mathematician, Von Neumann /J. Von Neumann Zur Theorie der Gesellschaftsspiele; Mathematische Annalen, Vol. 100, 3, 1928/ was taken up almost at once by representatives of the theory of limited usefulness (or by representatives of the econometric school), this does not attest to the uselessness or to a greater incorrectness of the mathematical theory of games itself.

In mathematics not only the initial idea of numbers and figures is being adopted from the real world, but also the formulation of all basic problems is dictated by human practices; and consequently it reflects this or that aspect of real phenomena of nature or of social life. In this respect the theory of games does not constitute something special. Abstractly speaking, even a game itself could as such serve as a topic for research. A chess theory exists, and so on. However, a game itself could become the occasion for a profound and complex mathematical theory only on the strength of the characteristics inherent in it, which are creating an interest going far beyond the limits which occupy the leisuretime of people. Games have been known since time immemorial. According to the evidence of some historians, games of hazard (dice) were already widely played in the world of antiquity. Chess also dates back many centuries. What accounts for the fact that games (of hazard) became an object for a mathematical theory comparatively recently — reputedly with the famous correspondence of Pasqual and Fermi, three centuries ago? This theory of strategic games, as has already been noted, goes back all of 3 decades.

One cannot attribute this fact to the lack of a body of mathematical concepts sufficient for this purpose. It is true that Laplace's analytical theory of probability could come to light only in the presence of an analysis of the infinitesimal, but Neumann's theory of strategic games requires the operativeness of the theory of great numbers for its generalizations. But the basic problem which had served as an occasion for the correspondence of Pasqual and Fermi as well as the much more simple problem of Neumann's theory (for example, a game of 2 opponents with a sum of zero), are resolved very simply.

Therefore the causes of the noted fact lie in something else. Games became scientific objects only when the analogical features became especially strikingly apparent in the practical activity of people. The theory of probability came to light because in it were reflected definite features of action, among them also social life, though these were treated only on the surface, but nevertheless completely objectively. However, quite soon after its birth the theory of probability began to conform to practical calculations connected with annuities with the insurance business and others more. Only an idealistic interpretation of the history of science can explain the origin of the theory of probability as a property of reason and not of reality, from the fact that bourgeois scientific thought of that period was not capable of seeing anything in its entirety, and that is why it turned toward the theory of probability as an anchor to safety, so long as in the complete absence of knowledge about each thing all there was to it was to say: "either it will be or it will not be, either it will rain, or it will snow." Of course, between the theoretical analysis of the surface of the phenomenon of chance and the opening up of the inner matter of social attitudes, a great chasm becomes apparent. But this does not make the theory of probability a less objective sector of mathematics than, let us say, the analysis of the infinitesimal.

In the classical theory of games everything depends on chance. Every possible outcome of a game has a definite probability, and each of them can occur; only the chance is in question. If some winnings are connected with each outcome (which could conceivably be a negative one, that is, a loss), that prize can thus turn out to be of a chance magnitude with a certain distribution of probability. We must remember that the game is "fair" if the stake put up to permit participation is equal to the mathematical expectation of a loss. In that case the average sum received in repeated games will be close to the average quantity of the stake put up.

Let us now return to strategic games which serve as the subject matter of Neumann's theory. Let us note first of all, that this theory does not speak about the discovery of special rules which the players must observe in chess (which, it seems, is the most typical example of a strategic game), or in dominoes, poker, and others. Such a theory would not be an abstract mathematical theory of games in general, but a special theory of a given game (like the long existing theory of chess), or a collective theory of a number of games. What is referred to here are general situations of games on the whole. According to this the strategic game differs from a game of hazard by the fact that the players, being confronted by a definite situation, have a choice of different actions, or moves. In a still more general manner, these actions which have been selected by the players are

intermittently interwoven with "chance moves," that is with changing situations which are purely accidental and not dependent on the choice of the players (such a combination is particularly typical, for instance, of all card games).

If one is to set up a table of previously selected moves for every possibility in a given game situation, such a list would form what is called "strategy" in the theory of games. It is clear that the result of each "meeting" is determined on one side exactly by all the strategic possibilities chosen by each player, and on the other side by the aggregate of the results of "chance moves." Having once and for all chosen a certain strategy, the chess champion would not have to trouble himself with traveling to tournaments; he could send his secretary to them whom he would have to provide with his strategy in, let us say, the form of an extensive card file containing all the possible situations and the moves chosen for each. Fortunately for those who love chess, the setting up of such strategy-cardfiles is in practice impossible, and they can in fact henceforth fully enjoy all the peripetia of this wonderful sport. Otherwise chess would really die out: either all the strategies used by the white pieces would result in victories against any of the strategies of the player with the black pieces (let us call this the "white-death" checkmate), or in the absence of this the white pieces would be moved by strategies which will at least not result in a loss against any opposition strategy (the notorious "no-man's death"); or it could be that it would signify the existence of a faultlessly executed strategy by the black pieces ("black death"). On the day when this question would be cleared up, the game of chess would lose all its meaning, and would at best be reduced to speculation; which of the players is to play with the white pieces and which with the black (the last deserves notice in view of what is to come). In games in which the players' choice is combined with the results of chance moves, it would preserve the meaning of the latter. It thus could be reduced to one speculation in which all its combinations are determined right away (with the probabilities of each combination determined according to the rules of the theory of probability, that is according to the theories of addition and multiplication).

It is already possible to observe the elements of strategy in the theory of games of chance if one has in mind not one but a whole series of matches. The players freely decide the question of how often to play and what bets to make (of course if they have something to bet). But in the theory of strategic games this is no longer a complicated secondary circumstance, but the decisive one. In strategic games, it appears, a large series of meets is taken for granted from the start because of the existence of chance moves, which permit the mathematical

expectation of winning to operate for each of the strategies, as if chance moves would not exist. The main thing is the idea of a battle between players, a battle of strategies.

We can show this battle on hand of a very simple example. First of all we have 2 participants. A pays B for the right to play the game, after which B pays A a sum whose mathematical expectation depends on the combination of strategies chosen by them, and which is defined in the following "table of payments":

		No of Strategies, B	
		I	II
No of Strategies, A	I	3	2
	II	0	1

Let A be the first to choose his strategy, which at once becomes known to B. Then after A has chosen I, B chooses II, so that he pays 2 instead of 3. After A chooses II, B chooses I, in order to pay 0, and not 1. Be is not able to make a better choice for himself and therefore choosing I, A ensures himself 2. This will not be the "price of a game" — this is a fee, which A must contribute before playing the game (the game does not lose its meaning because of this, since the chance moves which originate as a result of each round deviate on either side from 2). The decision which has been made corresponds to the "minimax principle" — the minimal maximum from the standpoint of B's strategies, or the maximal minimum for A. The essence of the contest lies in the fact that after A has made his choice, his opponent B attempts to counter in such a way that A's advantage is reduced to a minimum, and A must foresee this beforehand.

The examined case is called a case with a "saddle point."

Now imagine for yourself that the payment table is the following (this is not exact):

		No of Strategies, B	
		I	II
No of Strategies, A	I	3	1
	II	0	2

Now the outcome depends on who must make the first choice. If this is A, for example, the opponent then counters A's I with II (and pays the minimum, that is 1); and to A II he responds with I

(and pays 0). In view of this, A must choose I (so as to receive 1, and not 0). But if B makes the first choice, then he chooses II, which A must counter with the choice of II, and this is in contradiction to the foregoing, that is in contradiction to the fact that A chooses I for his first choice. Therefore, if there is no definite order of priority for the players and the combination of their strategy is to be determined by agreement between them (in advance, like the price in the act of buying and selling), such an agreement is here impossible.

In order for the game to be played, a different approach is possible: let the players determine their strategies, keeping them secret, and then inform any umpire, who announces the results according to their combination. At the same time, it is possible to entrust to him the drawing of chance moves. We will proceed to the chief specific conditions of the contemporary theory of games. How ought A to act?

It would seem that he must choose I in order to receive at worst 1, and not 0. But, putting himself in the place of B, he sees that on the basis of a related consideration B chooses II (in order to pay 2 at best, and not 3). But then it is better for A to choose II and receive 2 instead of 1. B, however, putting himself in his place, divines this reasoning and chooses I, on account of which A gets 0. But, understanding that B is choosing I, A chooses I, so as to receive 3. But B also sees through this and chooses II accordingly, so that A will receive only 1. A circle has been formed and it threatens endless repetition.

The theory of games answers the question thus: a system which must hold the attention of players exists in the absence of any system. Strategies must be determined by lot with some probabilities ("statistical determination"). In the example cited, A must play on fate either I or II with probabilities of $1/4$ or $3/4$.

It is now easy to define the validity of the combined strategies through the use of the theorem of multiplication:

	I B; $1/4$	II B; $3/4$
I A; $1/2$	$1/8$	$3/8$
II A; $1/2$	$1/8$	$3/8$

and to calculate the mathematical expectation of the outcome:

$$3 \cdot 1/8 + 1 \cdot 3/8 + 0 \cdot 1/8 + 2 \cdot 3/8 = 1.5$$

But if A deviates from the rules and begins for instance, to resort more frequently to I (in the hope for the large prize of 3) then, as soon as B notices this, he starts to apply his II strategy and A will more frequently get 1 instead of 2, that is, on the average less than 1.5.

What has remained from the strategies? In essence nothing: for now the umpire can only be entrusted with the technique of speculation of the strategy of A / B (or right away with the combination as indicated by probabilities), and everything is once more reduced to pure chance. The theory of strategic games has led to the decline of strategy. Only one difference remained in comparison with games of hazard: that the outcome is determined not only by the distribution of the probability of chance moves, but also by the distribution of the probability of strategy. Using the rules of addition and multiplication of probabilities, it is, however, easily possible to combine these distributions and reduce everything to one uniform lot for the determination of the outcome. It follows that the principal difference between the theory of strategic games and the theory of games of hazard is not so great.

Nevertheless, Neumann's theory of games has advanced our understanding of a series of correlations, namely on those occasions when the proper initial condition is present -- in practical conditions. This can best be seen from examples in the military field. Let, for instance, a plane hunt after a submarine and unerringly destroy it, if on a given occasion it turns up in one of its sectors, and there are only 2 sectors. If the staff of the airforce notices that the submarine has been in the habit of being active in A, then sending the plane there, it will destroy it. If the submarine commander notices that the plane searches for it all the time in A, he will operate with impunity in B.

If the submarine is going to be alternately in A and in B (let us say, on even days in A and on uneven days in B), this will easily be noticed and the plane will unerringly be sent to the spot where the boat is. Both opponents must choose a sector absolutely without any system, by drawing lots, which corresponds to the problem of the theory of games.

Similar problems are not difficult to point out in the capitalist economy with its competitive struggle. Two firms produce the same item in a week and send it either to London, or to Paris, where the weekly market capacity is the same. One firm has decided to ruin its competitor by cutting the price in half. But if it sends its item at this price to London, and if the competing firm at the same time went on the market in Paris, then its strategy and expenses were in vain;

it will be "selling too cheap" without a loss to the competitor. If this happens, where must the firm send its goods the following week? By simple reasoning, the aggressive firm, seeing that the competitor is active in Paris, also sends its goods there. But the competition catches on that an attempt is now being made to best it in Paris and sends the goods to London. The aggressive firm, however, putting itself in the place of the competitor, easily sees through its plan. But, putting itself in the other's place, the competition... Once again the task of such ugly and endless reasoning is being mastered by the "statistical decisions" of the theory of games.

It can now be clearly seen that the idea of such a struggle has been used as a source of inspiration by the authors of the theory of games. In the past, considerable time intervened between the first results of Pasqual and Fermi and the appearance of the theses about the theory of probability as the main key to the understanding of every action, particularly in social life. Everything happens more quickly in our century. In the first article about the theory of games, its author -- Neumann -- talks about card games and similar things, and observes only in a few words in a footnote that this also concerns the conduct of "economic man" -- this absolutely egoistic, ideally reasonable economic fellow. But this was picked up right away by bourgeois political economy. The basic work on the theory of games was published under the name of two authors: the mathematician Neumann was joined by the economist Morgenstern. J. von Neumann and O. Morgenstern; Theory of Games and Economic Behavior, Princeton, 1947. The first edition came out in 1943. But the theory of games as the key to economics had already been outlined in 1938 by E. Borelle, who had expressed such an idea as early as 1921, that is prior to Neumann's study of 1928 (see, for example, M. Frechet, Les mathematiques et le concret, Paris, 1955, p. 259). The theory of games was converted into the theory of economic behavior. The authors first of all stressed in it that its primary discovery consists of the rejection of the Robinson Crusoe-type man and in the substitution of the idea of struggle.

Competition also took place in Pasqual's time and during the entire history of pre-monopoly capitalism. But it required the extreme aggravation characteristic of the monopoly era to apply to it complex strategic plans, in order for it to be reflected in the founding of a whole new sector of mathematics. In the Neumann-Morgenstern theory the struggle of each against all -- bellum omnia contra omnes -- constitutes only the first part. The principal matter makes up the second part, in which are shown the conditions arising in games. The meaning of this stage is stressed by the authors themselves who show that the "contemporary" theory of games begins only with four and more players, since it is only thus possible to show that none of the players can partake outside a coalition. While perusing this second part, a reader feels as if he were turning a page in the book of economic history which separates the 19th from the 20th century.

The idea of a coalition gets its consistent development in the theory of games up to the inevitable formation of two universal coalitions of players, and the battle between them to which this leads. This is exactly equivalent to the much more terrible game of the capitalist coalitions at the start of our century. Everything in the world happened according to their laws. Now it is clear to us not only why games of chance had to wait a thousand years for a theory, but also why strategic games had to wait for another three centuries.

However, the direct cause for the theory of games being picked up by economists served also for something else. The subjective school of political economy, whose views make up the theoretical base of so-called econometry [The ideas of this school deserve a special analysis in themselves, but we are not aiming to do this here], found in the theory of games a way out of the difficulty which up to then had seemed to be insurmountable. This difficulty is most clearly apparent in the dynamic formulation of the problem. As is known, the main task of political economy in the bourgeois interpretation is the search for a point of balance between supply and demand, determined by lines of dependence of the one or the other on the price. If the supply in that case is unlimited the problem is reduced to the functional relationship between price and demand. Let us assume that we are talking about seasonal goods: fuel, let us say. The seller knows that the demand increases in the winter and he therefore calculates on raising the price in the winter. But, knowing this, the consumer (if he has the means) decides to lay in fuel in the summer, when it is cheaper. But the seller, divining these intentions of the consumer, raises the price in the summer. But then the consumer plans to stock up on fuel in the winter, etc., in the already well-known circle. In mathematics, the seller and consumer are like the plane and the submarine, and winter and summer are the two possible sectors of their activity.

Seeing in the theory of games a way out of a quandary, bourgeois political economy attempted right away to make it the main key to the understanding of all of economic life. The theory of games became an organic part of econometry (which had developed exactly from attempts at a mathematical solution of the tasks mentioned earlier). And it was quite fitting that the debated theory of games as the base for the theory of economic behavior was advanced by the Austrian (later American) economist O. Morgenstern, the eminent protagonist of the theory of limited utility, though he had first emphasized the difficulties of this theory which have been noted earlier.

The usual idealistic hypertrophy emerged from the theory of games in this contest. We saw that the capitalist economy has active features which are a reflection of it, but they are not the main thing. The competitive struggle is peculiar to capitalism, but it is not its main feature; exploitation is its main feature, and it is impossible to

comprehend the active role of competition without understanding that fact. Bourgeois consciousness exaggerates this aspect of capitalist economy, turns it into a major basis with an ingenious aim, as we will see below.

As soon as bourgeois economists took hold of the theory of games, it became for many of them the main basis for methodology. "What is our life? A game!", they began to repeat after Hermann in "Pique Dame." The economic man (homo economicus) is a rational egoist who is an active player in economics. The totality of his actions is his strategy. And as prize serves the sum total of happiness which he gets from his orders. It is true, this economic theory was enriched by a new hypothesis: that the "prospect" for a gain by either A or B with some probability is better than a simple gain by B, if only the usefulness of A is higher than the usefulness of B (and then, according to the theory of games, such a prospect can be estimated through mathematical estimates). How far removed such a hypothesis is from reality can be explained by any business executive, who is attempting to determine the price (a factory, for instance, prefers to prepare for making the worst cheese, than to know in general which cheese it will be able to dispose of, etc.). But without this hypothesis it is impossible to portray all of social life as a game, in which the rate of pay purchases the "prospect" with different possible results.

One can also see a number of other large difficulties in this matter. Of course, among them are those which already long ago have undermined the structure of the subjective school. Thus the hypothesis about the "transitivity" (in the language of mathematical theory) of the greatest number of lucky winners serves -- obviously or not -- as the initial imperative premise. It talks about the fact that from the stipulation "A's luck is greater than that of B, and B's luck is greater than that of C," the absolute conclusion must be drawn that "A's luck is greater than that of C"; and this alone cannot be proof, thus it becomes a base for the subjective school. One must accept this situation without proof, for if that is not done the theory of games cannot be applied to the lucky winner.

There are also specific difficulties which are especially peculiar to the interpretation of economics in the light of the theory of games; to overcome them one must generally rely on the serious contradictions with the old theory (called the "classical" for that reason). One must in particular assume that there exists an indivisible, general scale of utility without which the whole concept would collapse. For the subjective school, every explanation of barter and of everything that flows from it stems in the main simply from the fact that one and the same blessing is evaluated in different ways by the contractors. According to the basic principles of this school, no exchange could take place without such various evaluations. Meanwhile the "table of

payments" must be the only determining thing for the theory of games. This has to be assumed, so that in our table for the combination of both first strategies the winnings of 3 for A should not be viewed by B as a loss of 3 but, let us say, of 1, so that no single solution of the problem of selecting a strategy in accordance with the rules of the theory of game should become impossible. The attempts by Neumann and Morgenstern at the end of the book to avoid this difficulty by introducing some amendments to Bembaverk's formulation with regard to this matter comes to naught. As a result the interpretation of economics as a game turns it into an inevitable contradiction with its own basic principles.

But even more important is that which connects the peculiarities of the mathematical theory of games to the whole theory. The theory of games does not warrant such large hopes which are being put on it by virtue of its own mathematical properties. You see, it is totally insufficient for this, for in it some features of capitalist economy are reflected better (or in a more up-to-date manner), than in the classical theory. No extensive proof is required that the theory of games knows absolutely nothing of social life, ignoring the most important thing: the social order. And this is excluded from the theory of games precisely because the laws of social order have nothing to do with it, but with the rules of a factual and concrete "game." The life of a society and the interaction in it of its individuals is discussed in the theory of games out of context of time and in a vacuum, similar to how this theory discusses a chess game, not requiring even the knowledge of how many pieces are on the board. Equipped only with such an abstract theory, the chess player will suffer defeat at the very outset of the game. And the economist who tries to lean on this theory as the basis for his methodology will also suffer defeat.

It is easy to see that what becomes apparent here are the usual attempts of bourgeois science to find the key to the understanding of economics without a revelation of its internal contents, namely social relations.

To this have to be added a number of other much more particular considerations.

Even those phenomena which, it would seem, are best of all solved by the theory of games cannot be subordinated to its rules. Let us take an example where the very use of terminology points to a very close resemblance -- playing the stockmarket. A powerful capitalist "shark," selling short, let us say, puts a bundle of valuable papers on the market at the decisive moment, but this moment was not chosen by lot, like the theory of games requires, but by calculations involving concrete circumstances: the opponent's resources, the psychology of the great number of small investors, how strong the nerves of this group or another

are, etc. In order to be able to speak about the application of the theory of games it is necessary for the opponents to have unlimited opportunity to repeatedly observe each other's actions in related situations. But in economics, situations are non-repetitive, particularly in the previously mentioned stock-market "battles." In these the minimax principle is not reflected in the behavior which sets the tone of the powerful sharks; rather it has been true of the conduct of the dying out (if not extinct) type of investors like Timothy Forsythe, who prefers a small percentage from "sure" stocks.

Even more important is the fact that the theory of games investigates the main basis of economic life -- production -- most insufficiently. Evidently the Marxist interpretation of production as a struggle of society with nature has application here; this can even be shown in the terminology of the theory of games by some simple production problems. Thus let B in our first example of two strategies make the choice between two agricultural cultures, and let the numbers in the table represent the results achieved in each of two possible variations of a meteorological situation. If the harvest cannot be stored and if a half-starved existence requires a minimum result of 1, then the grower will prefer the second culture; even though it cannot give 3, it will provide at least 1 even in a poor year (the second strategy of A, whose role would be played by nature itself); whereas with a selection of the first culture a poor year could lead to death from starvation. It would seem that the producer acts in such a situation as in a game -- according to the minimax principle.

But in this case it is still not the specifics of the theory of games, not the situation which leads to a "statistical decision." In the example with the numbers on the second "payment" table introduced earlier, a year which is beneficial for the first culture is a poor one for the second, and the other way around. But nature is not a deliberate opponent who needs speculation in order to conceal the secret of his game; nature does not choose its strategy by calculating the strategy of the opponent in a manner which will reduce his results to a minimum. Because the statistical decision arrived at makes no difference we simply select the second culture in our example. If the product cannot be preserved for the next year, then the estimating of both cultures can be done simply by averaging the result and by calculating which part this or the other meteorological situation plays. Consequently, it is necessary to work in this case with probabilities and mathematical assumptions, but not quite with the same which are operative in the theory of games. In this case the decision will also include a simple choice of a culture with a large mathematical expectation of results, and it will not be of a statistical character. It is possible to present the matter differently, by simply providing nature with a consciousness which is hostile to man, somewhat in the way it was done with respect to the Shat mountains in the well known poetic legend by Lyermontov.

However, nothing ever came of the attempts to enlist the Kazbek mountains in a coalition against man.

One widespread misunderstanding must be noted here. As is known, the mathematical expectations of a result under given potential values (for instance in those payment tables) depends on the probability of these separate linear importances, that is one can consider them as the linearity of their functions. It is obvious that, having determined its "strategy" in production -- in our example the choice between cultures -- we find conditions which maximize the extent of such a linear function, whose aggregate of arguments is connected with them by a number of additional conditions. The maximum of the mathematical expectation of winnings as a linear function of probability finds its potential meaning also in the theory of games. This resemblance to a purely mathematical problem is engendered by some resemblance to a solution of the mathematical apparatus which it attracts. In the cited case such an apparatus turns out to be the so-called "linear programming," the significance of whose methods in the solution of a number of tasks which arise during the planning of production are not subject to any doubt. This serves as the cause for the fact that any problem which is being solved with the help of the methods of linear programming is attributed to the theory of games. Of course, in the theory of games it is also formally possible for one of the opponents to be afforded the opportunity to make only one "chance move" and to deprive him of any chance at "choice." But it can easily be seen that such a "game" is not a game in the spirit of the Neumann-Morgenstern theory. In any case it does not contain the most important thing, deliberately advanced in the first plan by the authors themselves: a battle of strategy between two opponents, that is in such a manner that their interpretation of economics contains something new in contrast to the independent-man concept repudiated by the authors. You see, the struggle with nature was also sufficiently clearly manifested in Robinson Crusoe's activities.

If it is true that the theory of games gave an additional, strong impetus to the development of linear programming, then the problems solved essentially with its help can be attributed to it with a basis which is no greater than any other mathematical problem in general, since in it, like in the theory of games, are found the applications of four rules of arithmetic.

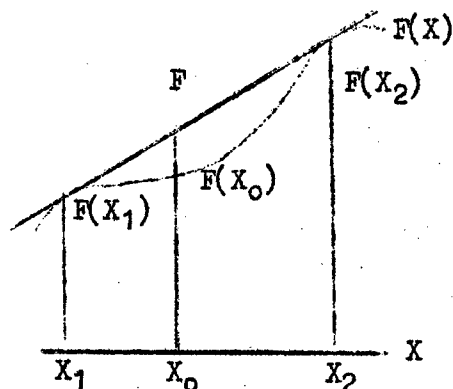
Even less resemblance to the original are found in a game which depicts production, with society as the other parties. In the Neumann-Morgenstern account it forms a single coalition in which all contradictions have disappeared. The common gains of the coalition need to be divided equally, in a brotherly way. Here we meet again, in a different aspect, and on another occasion, with the favorite bourgeois statistical index of average profit for all citizens without class differences, whose apologetic purpose it is not difficult to show. The

authors tolerate an additional game for the participants, to determine each one's share after a game in which all were combined against nature, only as a concession to economic activities. In that way, if the theory also concerns itself with problems of the distribution of profits, then this diligently isolates it from production. Meanwhile it is well known that it is exactly in production that attitudes arise which determine distribution.

Most important of all is the fact that the interpretation of social life as a game leads to apologetic sociological deductions. We are thinking here of such an acute problem as social inequality, its source and its consequences. In the light of the theory of games, social inequality is an inevitable result, since from any game some emerge as winners and others lose everything they have. The theory of games determines that success depends on the availability of information (in society that means education), on the ability to make correct choices on the basis of it (in society that means to be gifted), and finally on luck: on a combination of a strategy of the opponents determined by lot and chance moves. In society the latter means that in addition to all the talents and education of all there is also need for a little "luck," without which even the most talented and educated would perish as it is frequently possible to observe in the capitalist world; so frequently, that a theory which would not allow for the possibility of such phenomena would be in flagrant contradiction to the facts. Instead, everyone has the complete possibility to win and the complete freedom to play or not to play "prior to the game." If a part of the "players" is doomed to carry the whole burden of poverty, what is there to do: "Let the unlucky wretch pay!". Only let him curse his fate for it, and not the social order. Formally, capitalism affords the proletariat the freedom to die from hunger, but this does not apply to the owners of the means of production: don't play if you don't want to!

At this point one discovers that a very important link is missing in the contemporary theory of games. It determines the "price" for different games, that is, how much one must pay for the right to participate in them in one form or another. But it is necessary to have the wherewithal to pay. Besides education, talent and "luck" (information for the correct choice of strategies, success in chance moves) one other "trifle" is also needed for success in bourgeois society -- property. In the theory of games it is only a subjective factor of success and is not taken into account. It is necessary to give proper due to the old theory of games of hazard: in a number of problems (for instance, in a game leading to the complete ruin of one of the opponents, in some efforts at solving the famous paradox of the Petersburg game and others) this old theory was not afraid to draw attention to this factor (with respect to the starting capital of the players). It is easy to understand the historical causes of such a backward move.

One question still remains in this connection. If after a game the social-coalition against nature has gotten results in the form of some magnitude of profit above expectation, does that incite the people to partake in the above-mentioned "additional game" for distribution from which the participants emerge with a much more unequal share of the general profits of society? This is a question which has been posed and solved by the supporters of the theory of games as a theory of economic behavior. It has been proposed, let us say, for example by Freedman (Chicago) /"La theorie de l'incertitude et la distribution des revenus suivant leur grandeur," and a discussion on this paper in the collection "Econometrie," Paris, 1953, 7, that the usefulness or satisfaction which is being received from profit X constitutes some function emerging from it like $F(X)$, and that the average profit equals X_0 (in the subsequent account, by strictly confining ourselves to the principal aspect of the matter, we deviate considerably from the original for the sake of simplicity and brevity.). Though the theory of limited utility, which is being used as the starting point of all these constructions, usually assumes that the function of F increases slowly with an increase of X , the author however ascribes this to the fact that just in that section in which X_0 lies this function grows rapidly (see drawing). This rude



violation of the canons of the theory of limited utility remains on the conscience of the author, and even more so since it is well known that the agents of the capitalist economy are not aiming for any ephemeral "satisfaction" from profit, but for something considerably more realistic -- for the greatest amount of profit. Let us further imagine, that each individual can either be satisfied with the receipt of his X_0 , or demand instead a "perspective" with different possibilities; or saying it differently, to hurry with his X_0 to a gambling house to take part in the mentioned additional game. For simplicity sake let us say that the prospects which he is able to buy there consist of two possible values of profit, X_1 and X_2 (with the probabilities of P_1 and P_2). For this X_0 , which he had declined for the sake of such a game, society can

give him as a prospect only the mathematical expectation equal to X_0 , that is, $P_1 X_1 + P_2 X_2 = X_0$. But the prospect which the individual has given himself (the game) is estimated on the basis of the mathematical expectation of utility, and not on profit itself, that is on $P_1 \cdot F(X_1) + P_2 \cdot F(X_2)$. It remains to determine the maximum of this sum for a given X_0 in order to determine the X_1 and X_2 which with corresponding probabilities form an individual's prospects. If F started to increase slowly, then faster (and in this sector lies X_0), then again slowly, these X_1 and X_2 will be excisions [abscissa] of a point of contact which in two touches the straight line F in two places. (As a rule the relative maximum is reduced to $F'(X_1) = F'(X_2)$. If F simply increases quickly, then $X_1 = 0$ (the risk of losing everything!), but X_2 is in general unlimited. In the case of a slow growth or a uniform growth of F the problem simply loses its meaning.) For this probability P_1 and P_2 must be in the ratio $(X_2 - X_0) : (X_0 - X_1)$ -- as powers which lie at the two ends of a section with a given position for the center of gravity. This means that the more extensively the possible results of X_2 exceed the average profit X_0 , the less must be their probability.

In that way, if each changes the profit X_0 procured through brotherly sharing into the prospect of the mentioned probabilities of profit X_1 and profit X_2 , then each has the mathematical expectation of profit equal to X_0 . As long as many people participate in this, the average profit which they will realize from this will be equal to its mathematical expectation, that is to X_0 . But if it is everyone for himself, as the bourgeois economists who justify general egoism and inequality with theoretical arguments like to emphasize, everyone gets satisfaction from the very process of the game: there are even such amateurs who carry their savings into a gambling house, knowing beforehand that the game is "crooked" there, that is that the mathematical expectation of winnings is less there than the stake. It attracts them, however, because each hopes to become one of the few lucky ones who are fortunate at the expense of the rest. In this aspect the formulation under consideration has not yet introduced anything new. The new in it consists of the fact that under the stated conditions the judgment is made not on the basis of the size of the profit, but on the basis of the extent of the "satisfaction" received from it. For a profit of X_0 it is equivalent to $F(X_0)$. But when this profit is replaced in it by its mathematical expectation, then the satisfaction gotten from it is correspondingly replaced by the mathematical expectation F . As is quite apparent on the drawing, F is greater than $F(X_0)$ (since the mathematical expectation is the average of $F(X_1)$ and $F(X_2)$ with those probabilities in the character of weights. The apologists argue that owing to this, an individual exchanging his average profit for such a prospect gets not only pleasure from the game and hope for an overall good chance in it, but also a simple increase of anticipated satisfaction. For the whole aggregate of

individuals the expected average satisfaction turns out to be equal to its mathematical expectation, and since the latter is greater than satisfaction from an average return it means that the goal of society lies in a gain. It seems that an additional game is advantageous, since it permits the choice of the kind of prospect under which the mathematical expectation of utility is greater than the utility of the mathematical expectation (that is, X_0). The difference is a direct gain for each player and the average for society.

However, this gain exists for all only up to the game. But after it, a small segment of the players realizes a "large sum," equal to X_2 , but the majority is forced to limit itself to X_1 , that is, to a return considerably smaller than average. The average return now appears like the mathematical expectation which has existed for all, and the inequality arose from the general aspiration for maximum profit, for maximum utility. The additional game enticed everyone by virtue of the peculiarity of the false utility of the return which has been arbitrarily invented!

Such is the source of social inequality according to the theory of games. Its consequences are maximum social happiness: since the average magnitude of satisfaction has to coincide with its mathematical expectations, it is exactly such a situation which is the most desirable one for the purpose of society. In the Paris seminary where Friedman's lecture was read, some participants, congratulating the lecturer on his revelation, did not stint praise for the depicted social structure which affords everyone the possibility to play in such diverse and fascinating games.

Interest was also manifested in such details as the examined construction. If at the beginning there existed in society different groups with different given average returns of X_0' , X_0'' , etc., then all the individuals in each group could equally choose a prospect providing them with X_1 and X_2 (but with correspondingly different probabilities). As a result, according to the principle of absolute probability of the greatest number of recipients, X_1 and X_2 are obtained if one proceeds from the general average return for all groups together. The meaning of this detail consists of the fact that the division into classes means nothing, that the solution of the secret of inequality and the explanation of the numerical correlation of rich and poor lies in the theory of games and not in the class theory.

It would be possible to present a number of other details as well, but what has already been said is sufficient to show of what the unscientific, apologetic character of the theory of games as a theory of economic behavior and as the key to an understanding of the whole social life, consists. Is mathematics perhaps responsible for this? Of course not. We are not at all interested in a complete negation of

the contemporary mathematical theory of games which contains a number of interesting theses making it possible to investigate pertinent situations. It is above all impossible to deny the importance which this theory can have in the solution of certain problems in the sphere of military matters, and that alone is of considerable importance. The theory can also have a number of applications in other spheres and render considerable assistance in the analysis of some economic problems. But it is completely devoid of scientific foundations, as we have seen from the completely naive attempt to put the theory of games at the methodological basis of all social sciences. The fate of the theory of games in this respect is instructive. The mathematician, Neumann, well-known for a number of excellent studies in the sphere of the theory of diffusion and others, permitted himself to make imprudent mention in a footnote about economic man. It seemed that this was sufficient, for out of all of it, which is undoubtedly distinguished by harmony and refinement, the theory of games was turned into a weapon for apologists, and he himself became in some sense their scientific prisoner. Now it is necessary to spend considerable effort in order to liberate this theory from its apologetic utilization by economists of the subjective school. Would it not be better in the place of the mentioned imprudence to have stressed at the very outset the greatly limited sphere of its application?

The sphere of application of the idea of the theory of games cannot be the general methodological foundations of economic science. This does not mean that it cannot find application in the solution of specific problems, among them also economic ones. Clarity in this problem is important also for the expansion of the application of the mathematical apparatus which is being utilized in the theory of games, but not indissolubly connected with it, foremost in linear programming which need not be seen only through the eyes of the theory of games. It can have considerable economic effect in the determination of the optimal plan in transportation, the optimal variation of cooperation in enterprises, the distribution of tasks, the placing of industry, and in the solution of a number of other tasks. A particularly wide perspective opens up here in conjunction with the use of electronic machines. It is therefore most important for the economists to come up with a true Marxist theoretical-economic base for its application. Practical efforts in this direction have been made also by the economics faculty of the Moscow State University, and it is possible that in the near future their results can be imparted to the readers.